



**You have downloaded a document from
RE-BUS
repository of the University of Silesia in Katowice**

Title: Bound on the mass of Majorana neutrinos after SNO and KamLAND

Author: Agnieszka Joniec, Marek Zrałek

Citation style: Joniec Agnieszka, Zrałek Marek. (2004). Bound on the mass of Majorana neutrinos after SNO and KamLAND . "Acta Physica Polonica. B" (2004, no. 2, s. 639-646).



Uznanie autorstwa - Licencja ta pozwala na kopiowanie, zmienianie, rozprowadzanie, przedstawianie i wykonywanie utworu jedynie pod warunkiem oznaczenia autorstwa.



UNIwersYTET ŚLĄSKI
W KATOWICACH



Biblioteka
Uniwersytetu Śląskiego



Ministerstwo Nauki
i Szkolnictwa Wyższego

BOUND ON THE MASS OF MAJORANA NEUTRINOS
AFTER SNO AND KamLAND

A. JONIEC AND M. ZRAŁEK

Institute of Physics, University of Silesia
Uniwersytecka 4, 40-007 Katowice, Poland*(Received July 28, 2003)*

Presently the best terrestrial limit on light neutrino masses ($m < 2.2$ eV) are given by the tritium beta decay experiments. Not maximal mixing of solar neutrinos following from the SNO and KamLAND together with neutrinoless double beta decay $((\beta\beta)_{0\nu})$ data open the chance for better determination of the lightest of Majorana neutrino mass. We combine all available fits for the solar neutrino parameters and collect all Nuclear Matrix Elements (NME) calculations for the ^{76}Ge , nucleus for which presently the most stringent limit on the $(\beta\beta)_{0\nu}$ decay half-life time exist. We have shown that for some NME smaller bound on $(m_\nu)_{\min}$ can be found. Unfortunately one order of magnitude discrepancies in NME calculations do not allow to give the final answer.

PACS numbers: 14.60.Pq, 26.65.+t, 95.85.Ry

In the latest years the new atmospheric and solar neutrino experiments SuperKamiokande [1] and SNO [2] provide strong model independent evidence in favour of nonzero neutrino masses and mixing. All such oscillation type experiments determine a neutrino mass square differences $\delta m_{ij}^2 = m_i^2 - m_j^2$ and one is not able to find absolute values of their masses. As a precise determination of the oscillation parameters δm_{ij}^2 and $|U_{\alpha j}|$ is now only a matter of time, the determination of the neutrino masses m_i is very difficult and still wait for new more precise experiments or even for new experimental techniques. Knowing the δm^2 differences the problem of the masses alone is the problem of determination of the minimal mass $(m_\nu)_{\min}$. Also the other questions must be answered. First of all we should know the number of light massive neutrinos ($N_\nu = 3, 4$) and their nature (Dirac or Majorana?).

Several approaches both terrestrial and cosmological for measuring the neutrino masses are discussed in literature (see *e.g.* [3]). If neutrinos are

Majorana particles the neutrinoless double β decay should be observed [4] and the so called effective Majorana mass

$$\langle m_\nu \rangle = \left| \sum_{i=1}^n m_i U_{ei}^2 \right| \quad (1)$$

does not vanish. Many experiments on the search of $(\beta\beta)_{0\nu}$ decay were performed [5]. No indication in favour of such decay have been obtained up to now, and the upper bound on $\langle m_\nu \rangle$ was found. To be sure, an evidence of the $(\beta\beta)_{0\nu}$ -decay obtained from the reanalysis of the Heilberg–Moscow [7] experiment has been claimed [8], but it was strongly criticized [9]. The bounds on $\langle m_\nu \rangle$ combined with the results of neutrino oscillation can give important information about minimal neutrino mass $(m_\nu)_{\min}$ [10]. Quality of the information depends on value of the solar neutrino mixing angle θ_{solar} . Last KamLAND results [12] together with older solar neutrino data found that the value of θ_{solar} is far from giving maximal mixing ($\sin^2 2\theta_{\text{solar}} < 1$). That is the reason why we decided to analyze once more the limit for $(m_\nu)_{\min}$ which follows from the bound on $\langle m_\nu \rangle$. In the range of the present bound on $\langle m_\nu \rangle$ the normal ($\delta m_{\text{solar}}^2 = \delta m_{21}^2 \ll \delta m_{32}^2 \approx \delta m_{\text{atm}}^2$) and inverse ($\delta m_{21}^2 \approx \delta m_{\text{atm}}^2 \gg \delta m_{32}^2 = \delta m_{\text{solar}}^2$) mass schemes give similar results. In both cases we are in the almost degenerate mass region $m_1 \leq m_2 \leq m_3$ where the values of $\langle m_\nu \rangle_{\max}$ and $\langle m_\nu \rangle_{\min}$ can be easily found [11]

$$\langle m_\nu \rangle_{\max} \approx (m_\nu)_{\min} \quad (2)$$

and

$$\langle m_\nu \rangle_{\min} \approx (m_\nu)_{\min} (\varepsilon \cos^2 \theta_{13} - \sin^2 \theta_{13}), \quad (3)$$

where the ε parameter is defined by

$$\varepsilon = |1 - 2 \sin^2 \theta_{\text{solar}}|. \quad (4)$$

The formula (3) is valid for $\varepsilon > \tan^2 \theta_{13}$ which is satisfied for present values of the θ_{solar} and θ_{13} angles. From Eqs. (2) and (3) we see that the $\langle m_\nu \rangle_{\max}$ does not depend on the oscillation parameters and is explicitly defined by $(m_\nu)_{\min}$ only. In the same time the value of $\langle m_\nu \rangle_{\min}$ depends on $\sin^2 \theta_{\text{solar}}$ and the third mixing angle θ_{13} . This angle is small $\theta_{13} \approx 0$ so in practice $\langle m_\nu \rangle \approx \varepsilon (m_\nu)_{\min}$ and the solar mixing angle θ_{solar} decide about efficiency of the method of the neutrino mass determination. The method works better as θ_{solar} is farther away from the maximal value $\theta_{\text{solar}} \sim \pi/4$. In Table I we give all after KamLAND fits of the solar neutrino oscillation parameters $\delta m_{\text{solar}}^2$ and $\sin^2 \theta_{\text{solar}}$ (solar + KamLAND). The fits are generally not so much sensitive on $\sin \theta_{13}$, so we take here the old CHOOZ experiment limit,

TABLE I

Solar neutrino parameters.

	$\delta m^2 \times 10^5 \text{eV}^2$		
	best value	90% C.L.	99% C.L.
G.L. Fogli [14]	7.3	5.8-8.9	5.1-10.0
M. Maltoni [15]	6.9	5.8-8.8	5.0-9.9
J.N. Bahcall [16]	7.1	5.9-8.8	5.1-9.9
H. Nunokawa [17]	7.1	6.0-9.0	5.2-10.0
P. Aliani [18]	7.71	7.2-10.0	6.2-10.0
P.C. Holanda [19]	7.3	5.9-9.1	4.8-10.0
A.B. Balantekin [20]	7.1	6.0-8.8	5.2-9.9
A. Bandyopadhyay [21]	7.17	6.0-8.8	5.3-9.9
V. Barger [22]	7.1	5.4-9.1 (2σ)	>5.0 (3σ)
Mean value:	7.20	6.0-9.03	5.21-9.95
	$\sin^2 \vartheta$		
	best value	90% C.L.	99% C.L.
G.L. Fogli [14]	0.315	0.25-0.40	0.23-0.46
M. Maltoni [15]	0.315	0.24-0.405	0.225-0.46
J.N. Bahcall [16]	0.310	0.235-0.385	0.225-0.475
H. Nunokawa [17]	0.296	0.219-0.390	0.194-0.441
P. Aliani [18]	0.390	0.291-0.447	0.281-0.50
P.C. Holanda [19]	0.291	0.25-0.38	0.21-0.425
A.B. Balantekin [20]	0.315	0.25-0.41	0.23-0.46
A. Bandyopadhyay [21]	0.305	0.237-0.383	0.219-0.441
V. Barger [22]	0.296	0.24-0.35 (2σ)	0.22-0.39 (3σ)
Mean value:	0.315	0.246-0.394	0.226-0.450

$\sin^2 \theta_{13} < 0.04$ [13]. We see that the solar mixing angle is not maximal. In all six fits performed by different groups [14–22] the value of $\sin^2 \theta_{\text{solar}}$ even for 99% of C.L. is smaller than one. The mixing angle for the best fit, for 90% and 99% of C.L. in all papers are very similar. We calculate the average values, they are:

$$\text{best fit : } \sin^2 \theta_{\text{solar}} = 0.315, \quad (5)$$

$$90\% \text{ C.L. : } 0.246 \leq \sin^2 \theta_{\text{solar}} \leq 0.394 \quad (6)$$

and

$$99\% \text{ C.L. : } 0.226 \leq \sin^2 \theta_{\text{solar}} \leq 0.450. \quad (7)$$

In Fig. 1 we present the values of $\langle m_\nu \rangle_{\text{max}}$ and $\langle m_\nu \rangle_{\text{min}}$ as function of $(m_\nu)_{\text{min}}$. In the region of neutrino masses which we consider, the $\langle m_\nu \rangle_{\text{max}}$

is very simple (Eq. (2)) and independently of the solar mixing angle is given by one line (the thick solid upper line). The thick solid lower line correspond to the best fit value of mixing angles ($\sin^2 \theta_{\text{solar}} = 0.315$ and $\sin^2 \theta_{13} = 0$). The region between both these thick lines describes all possible values of the $\langle m_\nu \rangle$ where two Majorana CP violation mixing angles φ_i ($i = 1, 2$) change in the full domain $-\pi \leq \varphi_i \leq \pi$. As the limit on the neutrino mass depends on $\langle m_\nu \rangle_{\text{min}}$ we present the possible region of their values for the 90% (dark shaded) and 99% (light shaded) of C.L. for average values of the $\sin^2 \theta_{\text{solar}}$ (Eqs. (6),(7)) and $0 \leq \sin^2 \theta_{13} \leq 0.04$.

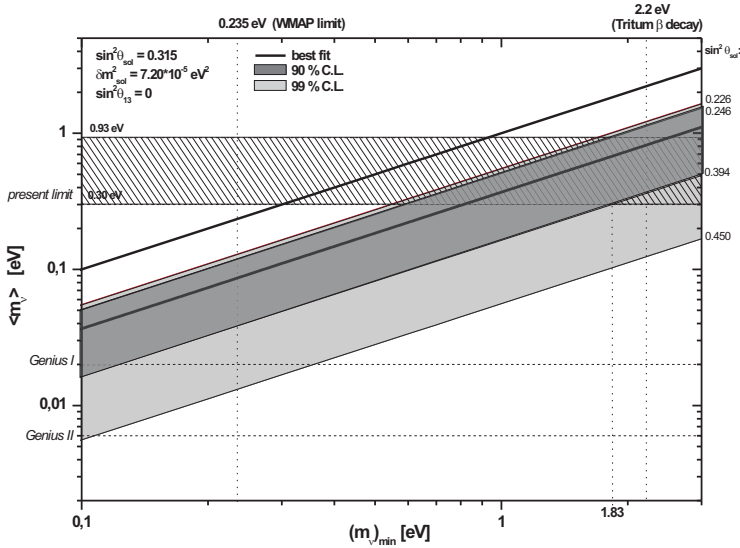


Fig. 1. The dependance of $\langle m_\nu \rangle_{\text{max}}$ and $\langle m_\nu \rangle_{\text{min}}$ on $(m_\nu)_{\text{min}}$. The thick solid upper line describes $\langle m_\nu \rangle_{\text{max}}$ and is independent on oscillation parameters. The thick lower line corresponds to $\langle m_\nu \rangle_{\text{min}}$ for the best fit values of solar neutrino parameters ($\sin^2 \theta_{\text{solar}} = 0.315$, $\sin^2 \theta_{13} = 0$). Possible values of $\langle m_\nu \rangle_{\text{min}}$ for 90% (99%) of C.L. are presented by the dark shaded (light shaded) region. Bounds on $(m_\nu)_{\text{min}}$ from tritium β decay and last WMAP limit [23] are also given. The present (upper hatched) and future (lower dashed lines) experimental and theoretical bounds on $\langle m_\nu \rangle$ are depicted.

In the linear relation between $\langle m_\nu \rangle_{\text{min}}$ and $(m_\nu)_{\text{min}}$ (Eq. (3))

$$\langle m_\nu \rangle_{\text{min}} = c(m_\nu)_{\text{min}} , \quad (8)$$

the slope c can be calculated, and $c = 0.37$ (for best fit), $c = 0.163$ (for 90% C.L.) and $c = 0.055$ (for 99% C.L.).

The experimental bounds on $\langle m_\nu \rangle$ ($\langle m_\nu \rangle_{\text{exp}} \leq \kappa$) decide about the upper bound on $(m_\nu)_{\text{min}}$. In the conventional left-handed coupling model, the $(\beta\beta)_{0\nu}$ rate is usually expressed as (see Ref. [24])

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} |M_{0\nu}|^2 \langle m_\nu \rangle^2, \quad (9)$$

where $G^{0\nu}$ is the phase space integral and $M_{0\nu}$ is Nuclear Matrix Element (NME). The determination of crucial parameter $\langle m_\nu \rangle$ from experimental results on $T_{1/2}^{0\nu}$ requires a precision knowledge of $|M_{0\nu}|^2$. Unfortunately it is complicated job and up to now there is no agreement between different methods of calculation.

For the isotope of Germanium ^{76}Ge the results which differ by one order of magnitude have been obtained

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) = b \times 10^{24} [y], \quad (10)$$

where (for $\langle m_\nu \rangle = 1$ eV)

$$\begin{aligned} b = & 1.7 [25], 2.16 [26], 2.3 [27], 2.33 [28], 3.15 [29], \\ & 3.2 [29], 3.6 [30], 4.06 [31], 8.95 [32], 14.0 [33], 17.7 [34]. \end{aligned} \quad (11)$$

Presently the best limit on $T_{1/2}^{0\nu}$ (^{76}Ge) have been found by Heidelberg–Moscow experiment [35]

$$T_{1/2}^{0\nu}(^{76}\text{Ge}) \geq 1.9 \times 10^{25} y \text{ (90\% C.L.)}, \quad (12)$$

which gives various bound on the $\langle m_\nu \rangle$ (90% C.L.)

$$\langle m_\nu \rangle < \sqrt{\frac{b}{19}} [\text{eV}] \in (0.30 - 0.93) \text{ eV}, \quad (13)$$

where only systematical errors are included. This region of possible bounds on $\langle m_\nu \rangle$ is also depicted in Fig. 1. We see that, in spite of new results on $\sin^2 \theta_{\text{solar}}$ the present bound on $\langle m_\nu \rangle$ and the present precision of NME calculation, give no chance to find better limit on neutrino mass. The bound (see Fig. 1) is larger than $m_\nu < 2.2$ eV [36, 37] given by tritium beta decay experiment. So the situation does not change in comparison with the previous searches [10, 11]. Now the problem is inherent in unprecise knowledge of the NME. In the optimistic scenario, if NME takes the large value (*e.g.* $b = 1.7$) then we have from (8) (for 90% C.L.)

$$(m_\nu) < \frac{\langle m_\nu \rangle_{\text{min}}}{0.163} = 1.83 \text{ eV}. \quad (14)$$

Unfortunately the present precision of NME calculation (11) does not allow to find the better limit on minimal neutrino mass.

Future oscillation experiments will diminish the errors of $\sin^2 \theta_{\text{solar}}$ and $\sin^2 \theta_{13}$, $\delta(\sin^2 \theta_{\text{solar}}) = 0.01$ and $\delta(\sin^2 \theta_{13}) \sim 10^{-4}$ [38]. Let us assume that their central values will not change very much, so we can expect that

$$\sin^2 \theta_{\text{solar}} \approx 0.31 \pm 0.01 \quad (15)$$

and

$$\sin^2 \theta_{13} \in (0, 10^{-4}) \quad (16)$$

giving the slope in Eq. (8)

$$c \in (0.36, 0.40) . \quad (17)$$

With the present bound on $\langle m_\nu \rangle$, the mass of the lightest neutrino will be still not well described

$$(m_\nu)_{\min} < \frac{1}{c} \sqrt{\frac{b}{19}} = (0.75 - 2.68) \text{ eV}. \quad (18)$$

The most sensitive experiments planned in the future will also use ^{76}Ge as a $(\beta\beta)_{0\nu}$ source. The GENIUS [39], Majorana [40] and GEM [41] experiments plan to reach the decay half-lives in the range 100×10^{26} y, 70×10^{26} y and 40×10^{26} y, respectively, giving a mass or a next bound for Majorana neutrinos in the range $(m_\nu)_{\min} \in (0.013 - 0.042) \text{ eV}$, $(m_\nu)_{\min} \in (0.016 - 0.050) \text{ eV}$ and $(m_\nu)_{\min} \in (0.021 - 0.067) \text{ eV}$. Even if obtained mass limits are much better then present and future tritium beta decay bounds, the uncertainties given by the NME are not pleasant.

So, in spite of the better determination of solar neutrino parameters still it is impossible to find the smaller bound on the lightest Majorana neutrino mass from the $(\beta\beta)_{0\nu}$ decay. However, on the contrary to the previous (after SuperKamiokande) search, such bound on $(m_\nu)_{\min}$ follows if we assume large NME for the ^{76}Ge decay. Unfortunately the one order of magnitude discrepancies in calculations of the NME, where various nuclear models are used, give presently no chance to find new bound on $(m_\nu)_{\min}$. Resolving the problem of NME's calculation for $(\beta\beta)_{0\nu}$ decay has now the crucial meaning.

REFERENCES

- [1] Y. Fukuda *et al.*, (SuperKamiokande Collaboration), *Phys. Rev. Lett.* **81**, 1562 (1998); Y. Fukuda *et al.*, (SuperKamiokande Collaboration), *Phys. Rev. Lett.* **82**, 2644 (1999); Y. Fukuda *et al.*, (SuperKamiokande Collaboration), *Phys. Rev. Lett.* **85**, 3999 (2000).
- [2] Q.R. Ahmad *et al.*, (SNO Collaboration), *Phys. Rev. Lett.* **87**, 071301 (2001); Q.R. Ahmad *et al.*, (SNO Collaboration), *Phys. Rev. Lett.* **89**, 011301 (2002); Q.R. Ahmad *et al.*, (SNO Collaboration), *Phys. Rev. Lett.* **89**, 011302 (2002).
- [3] H. Päs, T.J. Weiler, *hep-ph/0212194*; S.M. Bilenky *et al.*, *Phys. Rep.* **379**, 69 (2003).
- [4] W.H. Furry, *Phys. Rev.* **56**, 1184 (1939); M. Doi *et al.*, *Prog. Theor. Phys.* **70**, 1131 (1983); H.V. Klapdor-Kleingrothaus, H. Päs, A.Yu. Smirnov, *Phys. Rev. D* **63**, 073005 (2001); H. Nunokawa, W.J. Teves, R. Zukanovich Funchal, *Phys. Rev. D* **66**, 093010 (2002); S. Pascoli, S.T. Petcov, W. Rodejohann, *Phys. Lett. B* **558**, 141 (2003); H. Minakata, H. Sugiyama, *Phys. Lett. B* **532**, 275 (2002).
- [5] O. Cremonesi *hep-ex/0210007*.
- [6] H.V. Klapdor-Kleingrothaus *et al.*, *Eur. Phys. J. A* **12**, 147 (2001).
- [7] M. Günther *et al.*, (Heidelberg–Moscow Collaboration), *Phys. Rev. D* **55**, 54 (1997); L. Baudis *et al.*, *Phys. Rev. Lett.* **83**, 41 (1999).
- [8] H.V. Klapdor-Kleingrothaus *et al.*, *Mod. Phys. Lett. A* **16**, 2409 (2001); H.V. Klapdor-Kleingrothaus, *hep-ph/0205228*; H.V. Klapdor-Kleingrothaus, A. Dietz, I.V. Krivosheina, *Found. Phys.* **32**, 1181 (2002); H.V. Klapdor-Kleingrothaus, *hep-ph/0303217*.
- [9] F. Feruglio, A. Strumia, F. Vissani, *Nucl. Phys. B* **637**, 345 (2002); C.E. Aalseth *et al.*, *Mod. Phys. Lett. A* **17**, 1475 (2002).
- [10] S.T. Petcov, A.Y. Smirnov, *Phys. Lett. B* **322**, 109 (1994); S.M. Bilenky *et al.*, *Phys. Rev. D* **54**, 1881 (1996); S.M. Bilenky *et al.*, *Phys. Rev. D* **54**, 4444 (1996); J. Hellmig, H.V. Klapdor-Kleingrothaus, *Z. Phys. A* **359**, 351 (1997); H.V. Klapdor-Kleingrothaus, J. Hellmig, M. Hirsch, *J. Phys. G* **24**, 483 (1998); H. Minakata, O. Yasuda, *Phys. Rev. D* **56**, 1692 (1997); S.M. Bilenky *et al.*, *Phys. Rev. D* **54**, 6981 (1998); T. Fukuyama, K. Matsuda, H. Nishiura, *Mod. Phys. Lett. A* **13**, 2279 (1998); S.M. Bilenky *et al.*, *Phys. Lett. B* **465**, 193 (1999); G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos, *Phys. Rev. Lett.* **82**, 683 (1999); C. Giunti, *Phys. Rev. D* **61**, 036002 (2000); M. Czakon, J. Gluza, J. Studnik, M. Zrałek, *Acta Phys. Pol. B* **31**, 1365 (2000); S.M. Bilenky, S. Pascoli, S.T. Petcov, *Phys. Rev. D* **64**, 053010 (2001); H.V. Klapdor-Kleingrothaus, H. Päs, A.Yu. Smirnov, *Phys. Rev. D* **63**, 073005 (2001); S.M. Bilenky, S. Pascoli, S.T. Petcov, *Phys. Rev. D* **64**, 113003 (2001); Y. Farzan, O.L. Peres, A.Y. Smirnov, *Nucl. Phys. B* **612**, 59 (2001).
- [11] M. Czakon, J. Gluza, J. Studnik, M. Zrałek, *Phys. Rev. D* **65**, 053008 (2002).
- [12] K. Eguchi *et al.*, (KamLAND Collaboration), *Phys. Rev. Lett.* **90**, 021802 (2003).
- [13] M. Appolonio *et al.*, *Phys. Lett. B* **466**, 415 (1999).

- [14] G.L. Fogli *et al.*, *Phys. Rev.* **D67**, 073002 (2003).
- [15] M. Maltoni, T. Schwetz, J.W.F. Valle, *Phys. Rev.* **D67**, 093003 (2003).
- [16] J.N. Bahcall, M.C. Gonzales-Garcia, C. Pena-Garay, *J. High Energy Phys.* **0302**, 009 (2003).
- [17] H. Nunokawa, W.J.C. Teves, R. Zukanovich Funchal, *Phys. Lett.* **B562**, 28 (2003).
- [18] P. Aliani *et al.*, *Phys. Rev.* **D68**, 053001 (2003).
- [19] P.C. de Holanda, A.Yu. Smirnov, *JCAP* **0302**, 001 (2003).
- [20] A.B. Balantekin, H. Yuksel, *J. Phys. G* **29**, 665 (2003).
- [21] A. Bandyopadhyay, S. Choubey, S. Goswami, *Phys. Rev.* **D67**, 113011 (2003).
- [22] V. Barger, D. Marfatia, *Phys. Lett.* **B555**, 144 (2003).
- [23] D.N. Spergel *et al.*, *Astrophys. J. Suppl. Ser.* **148**, 175 (2003).
- [24] M. Doi, T. Kotani, E. Takasugi, *Prog. Theor. Phys. Suppl.* **83**, 1 (1985).
- [25] W.C. Haxton, G.J. Stephenson Jr., *Progr. Part. Nucl. Phys.* **12**, 409 (1984).
- [26] T. Tomoda *Phys. Prog.*, **54**, 53 (1991).
- [27] J. Engel *et al.*, *Phys. Lett.* **B225**, 5 (1996).
- [28] A. Staudt *et al.*, *Europhys. Lett.* **13**, 31 (1990).
- [29] A. Faessler, F. Simkovic, *J. Phys. G* **24**, 2139 (1998).
- [30] G. Pantis *et al.*, *Phys. Rev.* **C53**, 695 (1996).
- [31] J. Suhonen, S.B. Khadkikar, A. Faessler, *Nucl. Phys.* **A529**, 727 (1991);
- [32] Šimkovic *et al.*, *Found. Phys.* **27**, 1279 (1997).
- [33] J. Engel *et al.*, *Phys. Rev.* **C37**, 731 (1988).
- [34] E. Caurier *et al.*, *Nucl. Phys.* **A654**, 973 (1999).
- [35] H.V. Klapdor-Kleingrothaus *et al.*, *Eur. Phys. J.* **A12**, 147 (2001).
- [36] J. Bonn *et al.*, *Nucl. Phys. Proc. Suppl.* **91**, 273 (2001).
- [37] V.M. Lobashev *et al.*, *Nucl. Phys. Proc. Suppl.* **91**, 280 (2001).
- [38] A.G. Cocco, (OPERA Collaboration), *Nucl. Phys. Proc. Suppl.* **85**, 125 (2000);
K. Lang, (MINOS Collaboration), *Nucl. Instrum. Methods* **A461**, 290 (2001);
F. Cavanna *et al.*, (ICARUS Collaboration), LNGS-P21-99-ADD-2.
- [39] H.V. Klapdor-Kleingrothaus *Int. J. Mod. Phys.* **A13**, 3953 (1998),
H.V. Klapdor-Kleingrothaus, J. Hellmig, M. Hirsch, *J. Phys. G* **24**, 483 (1998),
H.V. Klapdor-Kleingrothaus *et al.*, *hep-ph/9910205*.
- [40] C.E. Aalseth *et al.*, (MAJORANA Collaboration), *hep-ex/0201021*.
- [41] Y.G. Zdesenko, O.A. Ponkratenko, V.I. Tretyak, *J. Phys. G* **27**, 483 (1998).